Sample Proof

Theorem 1. *Let a and x be elements of a group G. The orders of the elements a and (xax^{-1}) are the same.*

Proof. Let $|a| = n$.

To show that the order of (xax^{-1}) is also n we need to show:

1. $(xax^{-1})^n = e$, and
2. $(xax^{-1})^m \neq e$ for any $0 < m < n$.

Then we can conclude that $|xax^{-1}| = n$.

The first part is straightforward:

$$\begin{aligned}
(xax^{-1})^n &= \underbrace{(xax^{-1})(xax^{-1})\dots(xax^{-1})}_{n \text{ terms}} \\
&= xa^n x^{-1} && \text{by reassociating and cancelling the } x\text{'s} \\
&= xex^{-1} && \text{since } |a| = n \\
&= xx^{-1} \\
&= e
\end{aligned}$$

We will prove the second part by contradiction. In order to arrive at a contradiction, we assume that $0 < m < n$ and $(xax^{-1})^m = e$.

As above, we know that

$$\begin{aligned}
e &= (xax^{-1})^m \\
&= xa^m x^{-1}
\end{aligned}$$

We then solve for a^m to get

$$\begin{aligned}
a^m &= x^{-1}x \\
&= e
\end{aligned}$$

This contradicts the fact that $|a| = n$ since m is a positive number smaller than n. Thus we have proven the second assertion that $(xax^{-1})^m \neq e$ for any $0 < m < n$.

Therefore, $|(xax^{-1})| = n$, and $|(xax^{-1})| = |x|$.

□