

Nim: The Game

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Origins of Nim

- ▶ "Original" Rules come from Nim, A Game with a Complete Mathematical Theory by Charles Bouton. Published in The Annals of Mathematics in 1902.
- ▶ Nim comes from the German verb "nehem" which means "to take."
- ▶ Many sources indicate Nim originated in China and was called Fan Tan. Actually it was Tsyanshidzi or "picking stones game."
- ▶ It appeared in Europe in the 15th century.

"Original" Rules

- ▶ Two players
- ▶ Three piles of objects. The piles can be any size as long as no two are equal.
- ▶ Players alternate turns.
- ▶ In any move, the player must remove at least one object from a pile of his or her choice. An entire pile could be removed in one turn. Objects from different piles cannot be removed in the same move.
- ▶ The player who takes up the last object or objects wins.

General Nim Rules

- ▶ Two players.
- ▶ Arbitrary number of piles, each pile contains an arbitrary number of objects.
- ▶ Players alternate turns.
- ▶ In any move, the player must remove at least one object from a pile of his or her choice. An entire pile could be removed in one turn. Objects from different piles cannot be removed in the same move.
- ▶ The player who takes up the last object or objects wins.

Uses for Game Theory

- ▶ Economic problems
- ▶ Political Science
- ▶ Psychology
- ▶ Sociology
- ▶ Marketing
- ▶ Finance
- ▶ Warfare

Features of Combinatorial Game Theory

1. There are two players who alternate turns.
2. The game is deterministic.
3. There is perfect information.
4. Play must end.
5. The last move determines the winner.
 - ▶ Normal play - the last player to move wins.
 - ▶ *Misère* play - the last player to move loses.

Importance of Nim

Sprague-Grundy Theorem shows that every position in every impartial game is equivalent to a position in Nim!

How to Win (Original Version)

- ▶ Safe combinations
 - ▶ Write the number of objects in each heap in base two.
 - ▶ Place these numbers in three horizontal lines so that the units are in the same vertical column.
 - ▶ Add these base two numbers "without carry"
- ▶ If this sum is zero then your game is in a safe combination.

Taylor's Problem

0	1	1	0	x
0	1	0	x	1
1	1	x	1	1
0	1	0	1	0
	0	1	0	0

- ▶ Add both the columns and the rows giving us a column of numbers and a row of numbers. We want to know when these sums are symmetric.
- ▶ If the sums are symmetric, then the x 's on the diagonal can take on any acceptable value and the sums will remain the symmetric.

Variations on Nim

- ▶ Matrix Nim
- ▶ Wythoff Nim
- ▶ Global Nim
- ▶ Nim with a Modular Muller Twist

Other questions about Nim

- ▶ What does changing the initial number of heaps do to the game?
- ▶ What does looking at prime factorizations of number of objects, and how those are arranged in heaps do to the game?
- ▶ Can we find the smallest number of moves until the game is won?
- ▶ What happens when we use different bases to investigate strategy?

Sources for this Talk

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